2018 年度日本政府（文部科学省）奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2018

学科試験問題
EXAMINATION QUESTIONS

高等専門学校留学生
COLLEGE OF TECHNOLOGY STUDENTS

数学

MATHEMATICS

注意　☆試験時間は 60 分

PLEASE NOTE: THE TEST PERIOD IS 60 MINUTES

	Nationality		No.				
MATHEMATICS	Name	(Please print full name, underlining family name)			Marks		

1   Answer the following questions and write your answers in the boxes provided.

1) Solve the equation $x^3 + x^2 - 4x + 2 = 0$.

$$x =$$

2) Solve the equation $\cos 2x + 3\cos x + 2 = 0 \quad (0 \le x < 2\pi)$.

$$x =$$

3) Solve the equation $3^{2x+1} + 5 \cdot 3^x - 2 = 0$.

$$x =$$

4) Solve the inequality $4^{x+1} + 11 \cdot 2^x - 3 \ge 0$.

5) Solve the equation $(\log_2 x)^2 = \log_4 x^4$.

$$x =$$

6) Solve the inequality $\log_3(3 - x) + \log_3(x + 1) < 1$.

7) Let $\vec{a}$ and $\vec{b}$ be two vectors such as $|\vec{a}| = 1$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 2$. Calculate $|2\vec{a} - 3\vec{b}|$.

$$|2\vec{a} - 3\vec{b}| =$$

8) The line l passes through the intersection point of the line $7x - y = 5$ with the line $3x + 2y = 7$. The line l is perpendicular to the line $x - 2y - 3 = 0$. Find the equation of the line l.

9) The Nth partial sum S_N of the sequence $\{a_n\}$ satisfies the following condition. Find the nth term a_n of the sequence $\{a_n\}$.

$$S_N = 3^N + 2N - 1.$$

$$a_n =$$

10) Calculate $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 3x + 4} - x\right)$.

11) Let $f(x) = \log_e\{x(x + e)\}$. Calculate $f'(e)$.

$$f'(e) =$$

12) Calculate $\displaystyle\int_0^{\frac{\pi}{2}} x\cos x \, dx$.

2 Let $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} a & -2 \\ 1 & 2 \end{pmatrix}$, and $C = \begin{pmatrix} b & 2 \\ -1 & -1 \end{pmatrix}$ satisfying the following condition. Answer the following questions and write your answers in the boxes provided.

$$B^2 = B, \ BC = CB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

1) Find a and b.

$a =$ $b =$

2) Suppose $A = xB + yC$. Find x and y.

$x =$ $y =$

3) Find A^5.

$A^5 = \begin{pmatrix} & \\ & \end{pmatrix}$

3 Let $f(x) = \dfrac{\log_e x}{x}$ $(x > 0)$. Answer the following questions and write your answers in the boxes provided.

1) Find the maximum value M of $f(x)$.

$$M =$$

2) Find the tangent line l to the curve $y = f(x)$ passing through the point $(0, 0)$.

3) Calculate the area S among the curve $y = f(x)$, the line l, and the x-axis.

$$S =$$